

# Modeling GPS Positioning Errors due to Ionospheric Scintillation



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# Outline

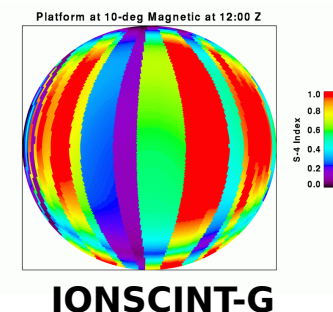
- Program components
  - Bench-top simulation with SN (WPAFB)
  - Waveform simulation (data, models)
  - Modeling GPS receiver errors
  - Modeling GPS system impacts (SN)
- Characterize scintillation-induced GPS positioning errors from actual data
- Principal scintillation effects on GPS navigation
- Ad hoc model description and preliminary results



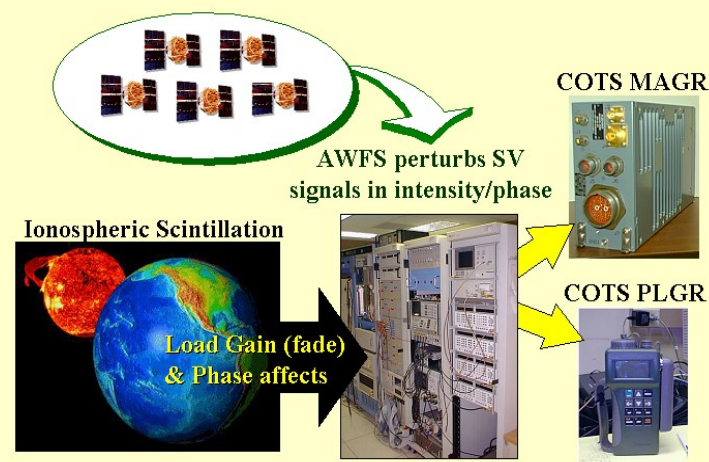
# GPS Modeling & Simulation Capabilities: The Right Tools for the Job

Neither cost-effective nor feasible to perform  
comprehensive field tests on all GPS UE

- Essential components for high fidelity GPS simulations exist
  - Complex scintillation waveforms (VS Hanscom)
  - “Hardware-in-the-loop” wavefront simulation capability (SN Wright-Patterson)
  - Battlespace scintillation scenario generation and simulation (VS Hanscom; SN Wright-Patterson; ...)
- Upgrades to waveform generation and wavefront simulation technologies required
  - Improvements in resolution, phase control, etc. required for SNRW capability
  - VSBX enhancements to waveform generation
- Goal to produce dual-frequency GPS nav error products tailored to operational UE achievable



## SNRW Hardware Simulator



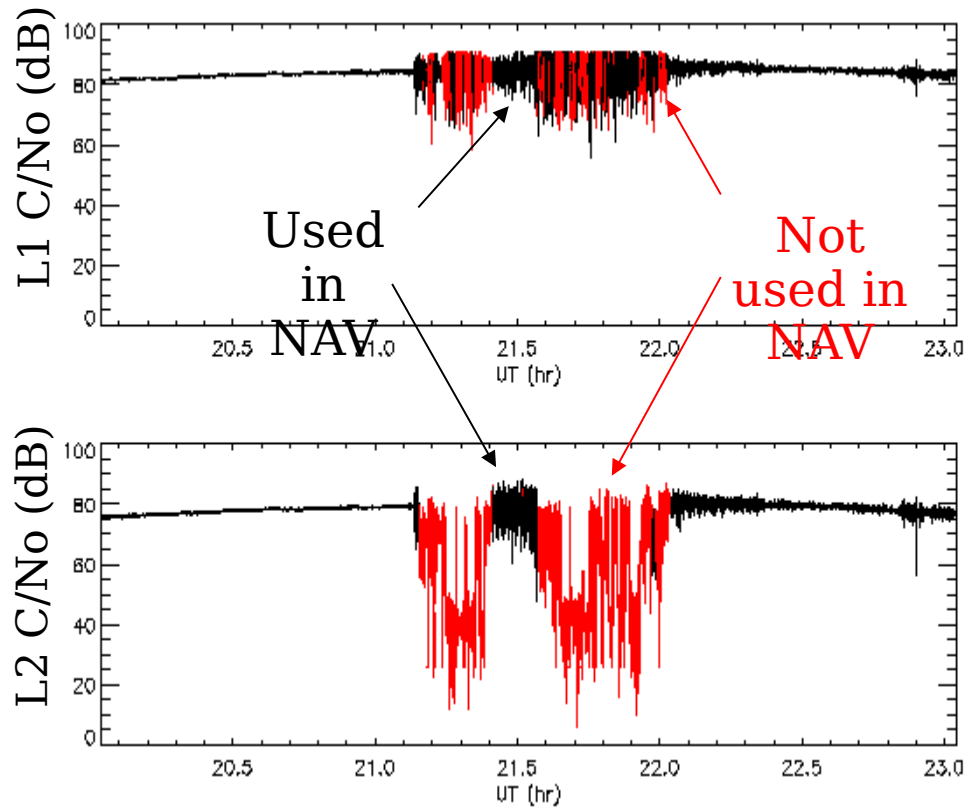
Investment in existing M&S technology needed



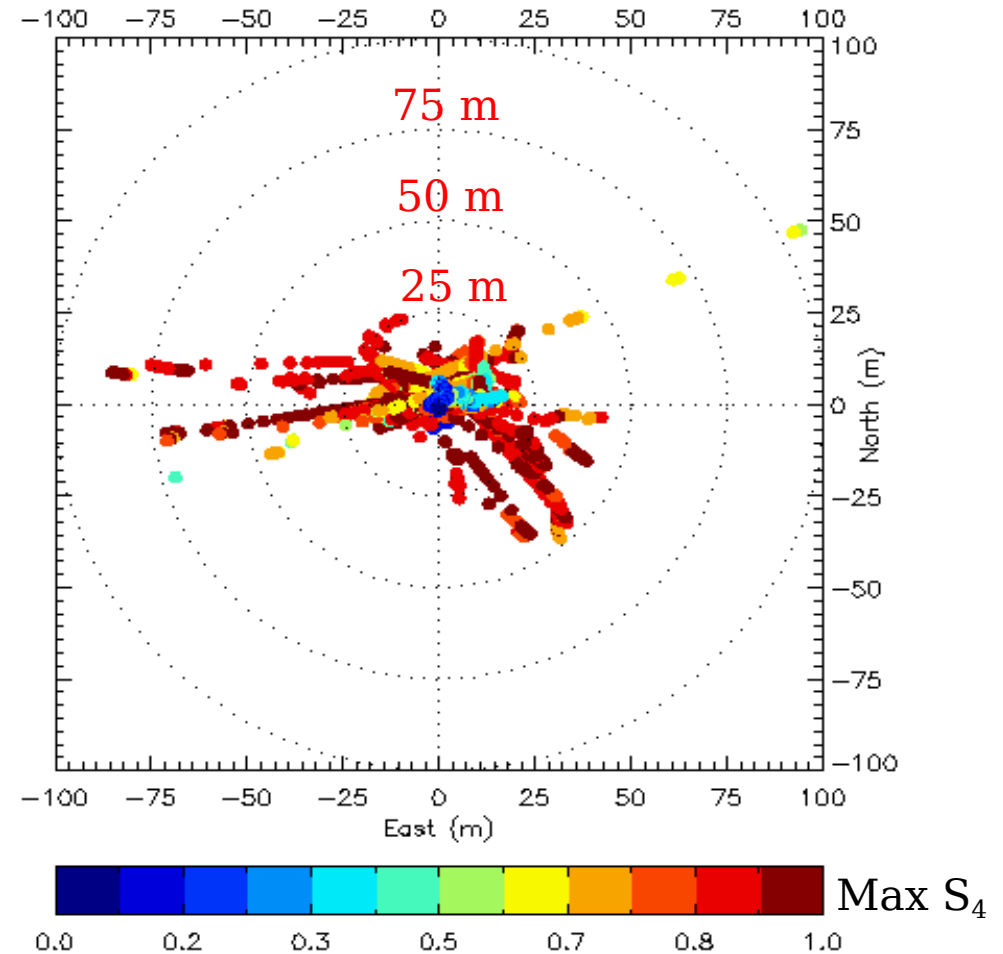
# Scintillation Effects on Positioning Accuracy



16 Mar 2002, ASI



Scintillation Causes **Fading** of L1 and L2 GPS Signals

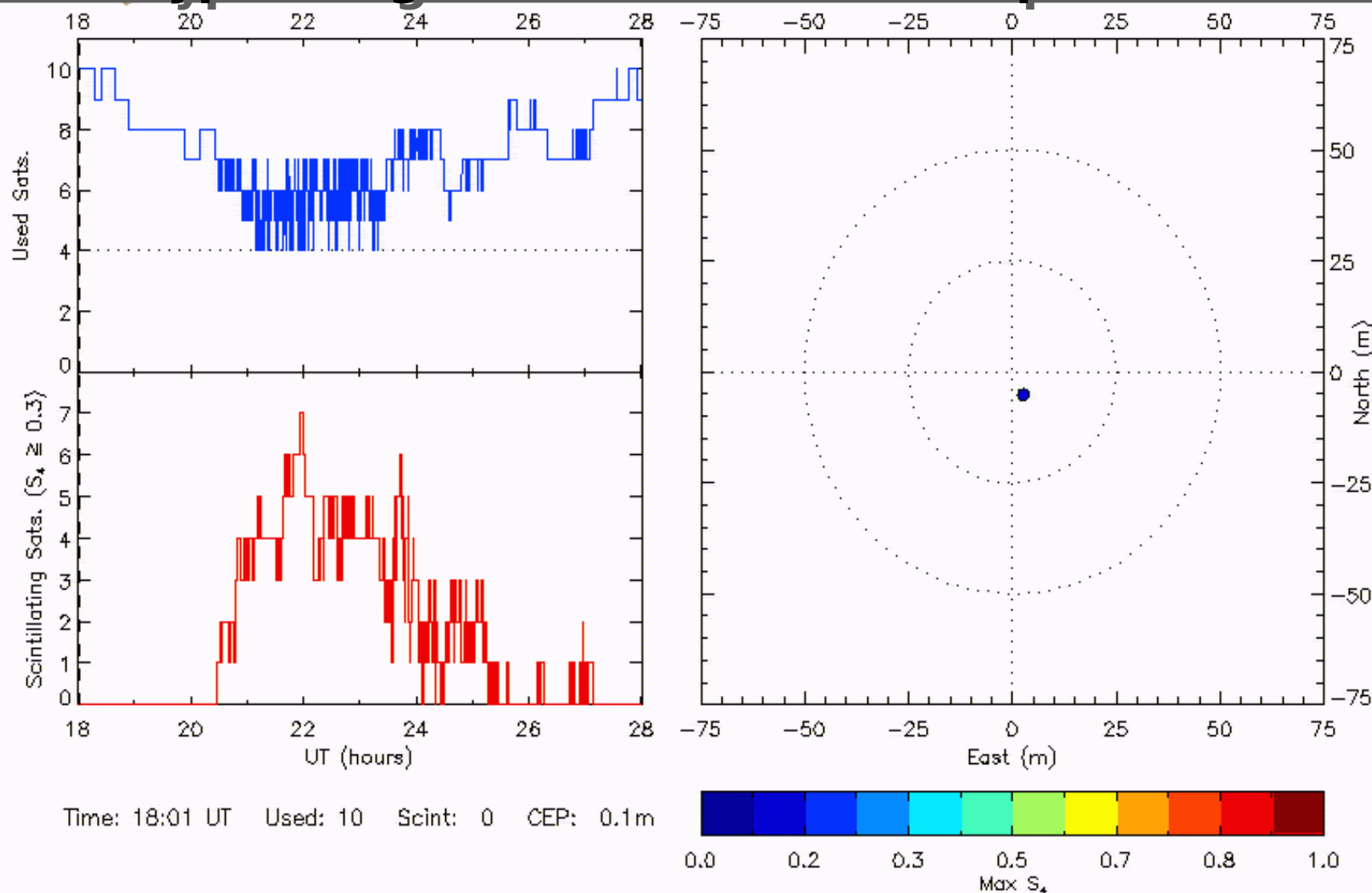


Resulting Positioning Error



# GPS Positioning Errors During Solar Max

**Scintillation can cause rapid fluctuations in GPS position fix;  
Typical night from recent field experiments**



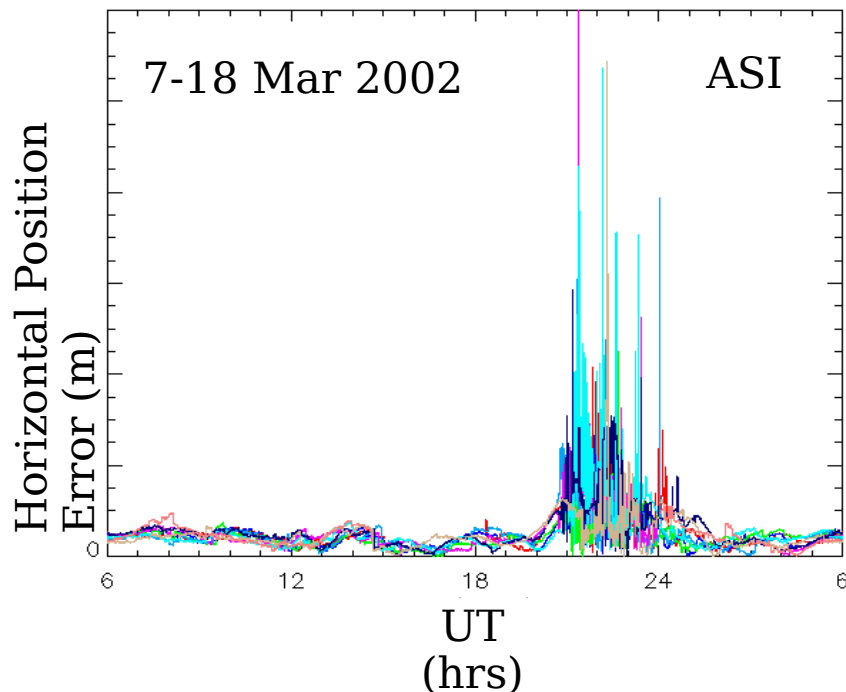


# Representative Positioning Errors Near Solar Maximum

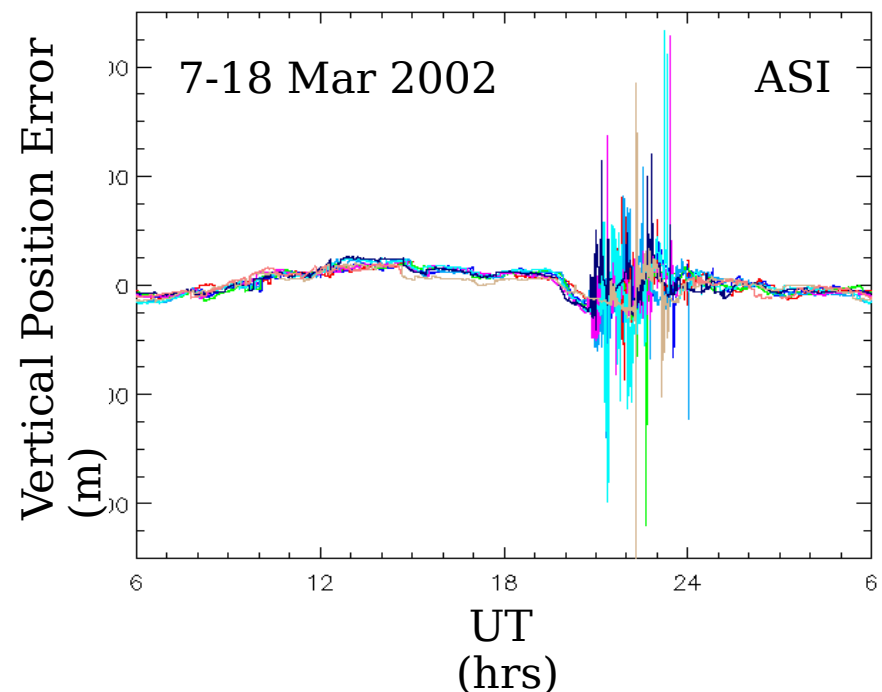


Position from dual  
frequency receiver  
with access to  
encrypted Y code

Active Ionosphere  
21:00-23:30 UT



Horizontal Error > 100 m



Vertical Error > +/- 200 m



# Statistical Analysis of Positioning Errors

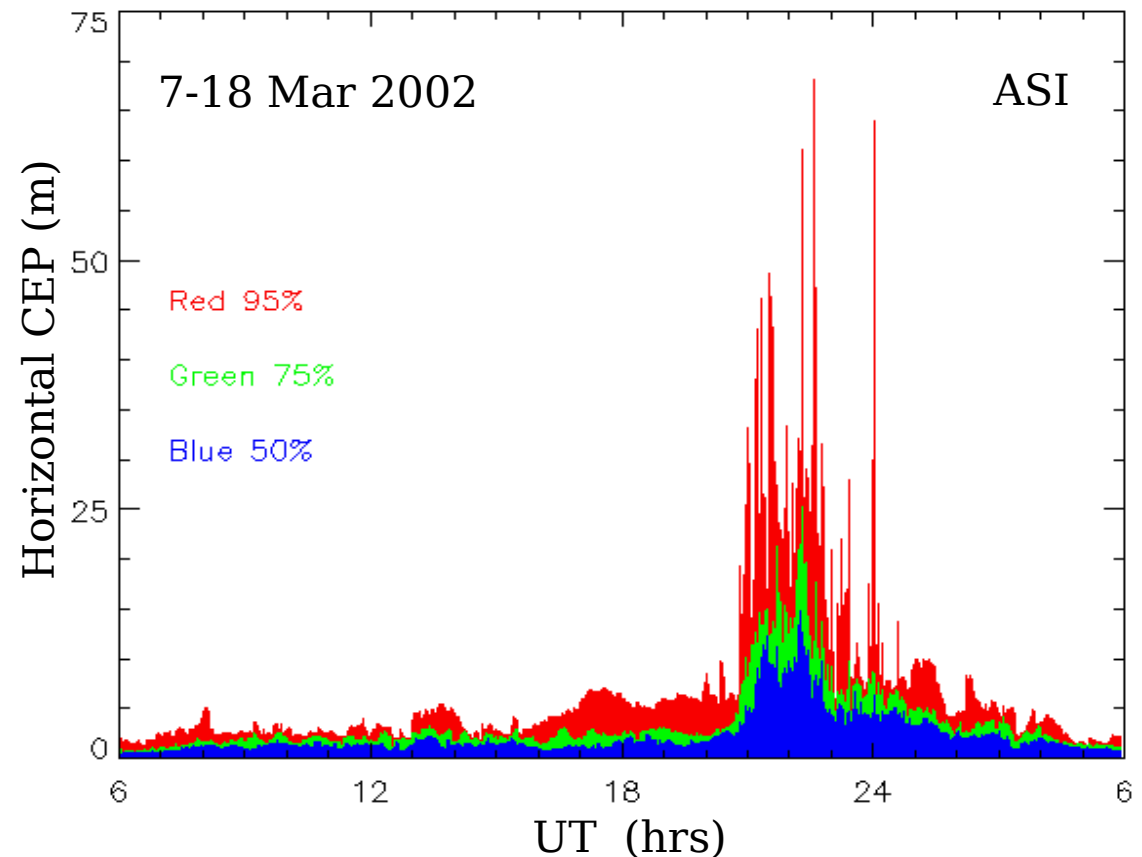


## Circular Error

### Probability:

probability that error will exceed a given level

A possible metric for a position error product



Single point positioning error (2D) better than 10 meters 95% of the time ... except during scintillation





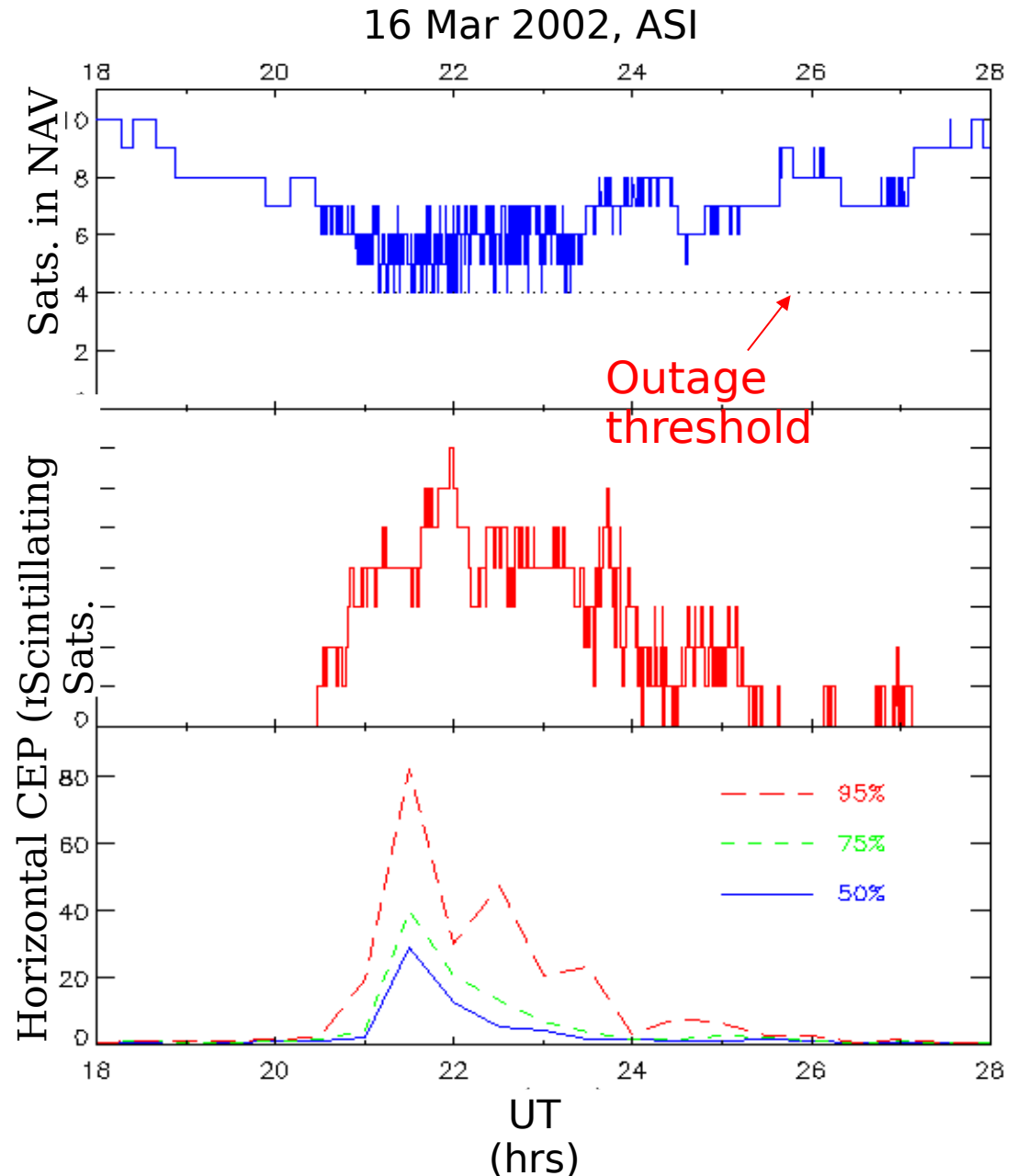
# Scintillation Degrades Effective Geometry of the GPS Satellite Constellation



Scintillation causes  
“Intermittent  
availability”

**All** visible satellites  
were scintillating at  
22 UT  
(not uncommon at  
ASI)

Position errors grow  
as  
number of available  
satellites is reduced







# Geometrical Errors and Ranging Errors



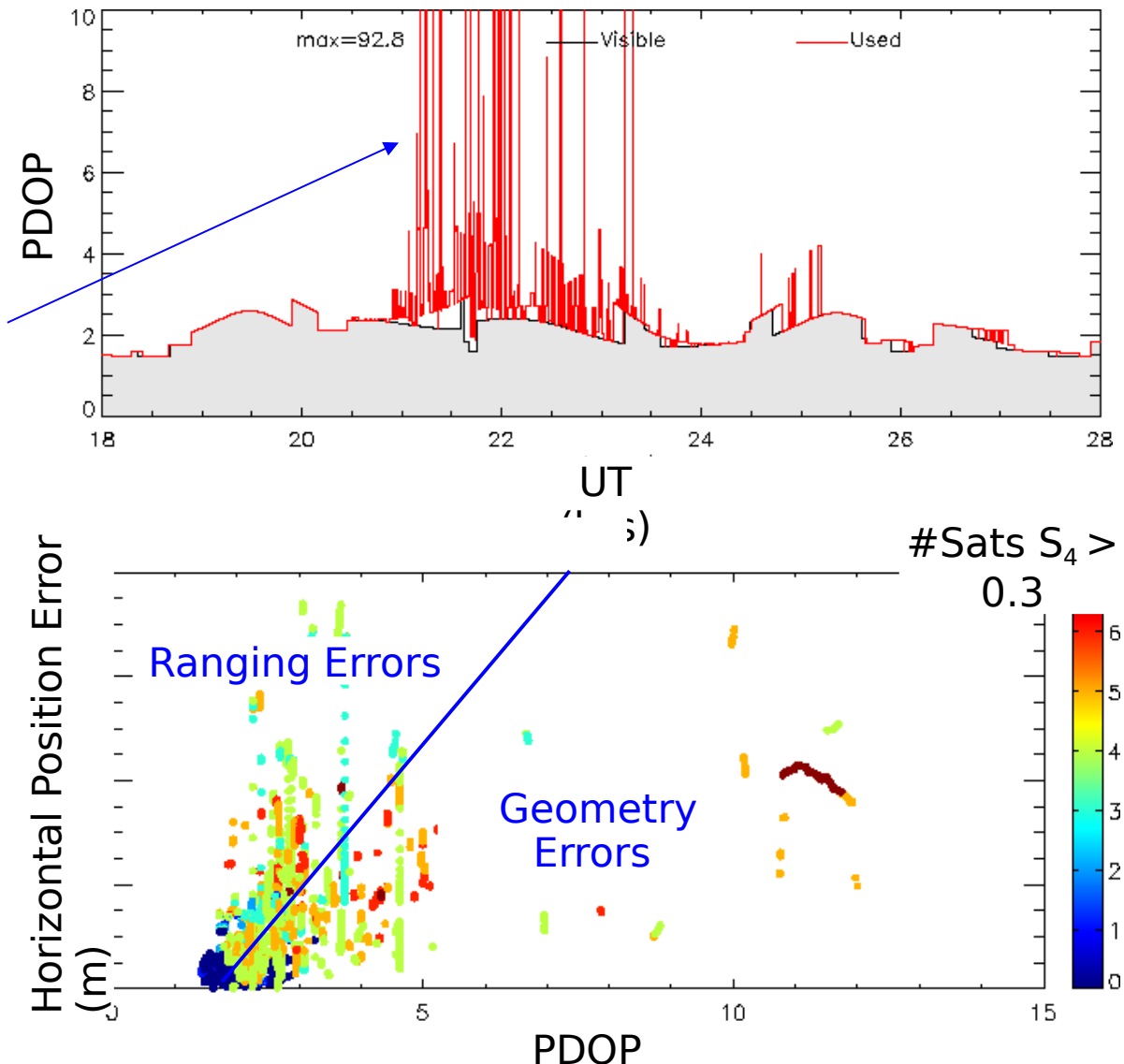
Theoretical and **measured** Dilution of Precision (DOP)

**Spikes** occur when a satellite becomes temporarily unavailable (timescale ~ seconds, dictated by satellite and plasma velocities)

Large DOP generally leads to large errors, but ... position error can be large even when DOP is good

(**>70 m with PDOP of 3**)!

Conclusion:  
scintillation causes  
**ranging errors**





# Modeling the Effects of Scintillation on GPS

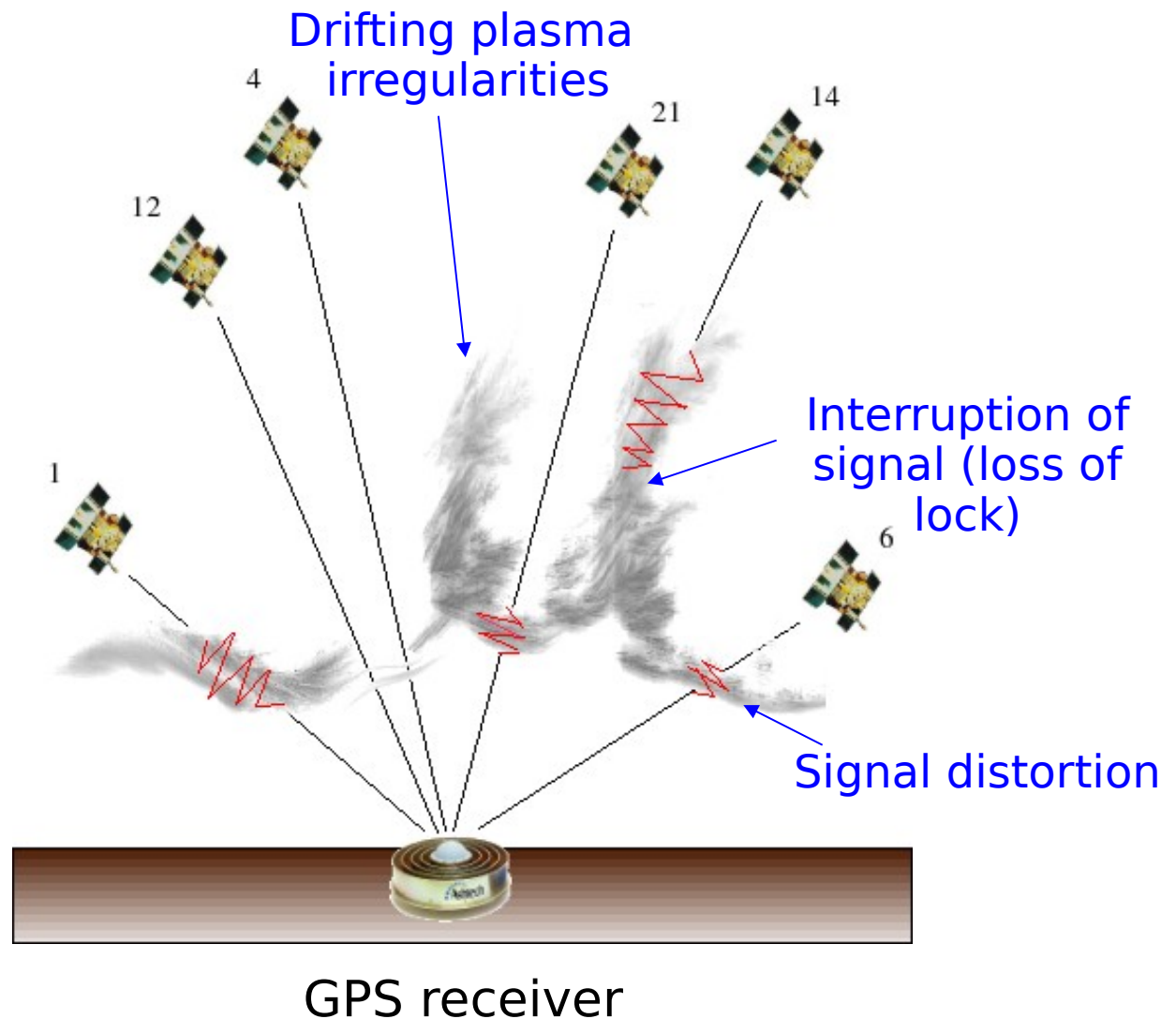


Two **principal effects** to model:

- Intermittent availability
- Induced ranging errors

For realistic predictions, model must couple these effects with the **constellation geometry**

e.g. distortion of signal from a lone zenith satellite much more damaging than from a redundant mid-elevation satellite





# Modeling GPS Satellite Availability During Scintillation



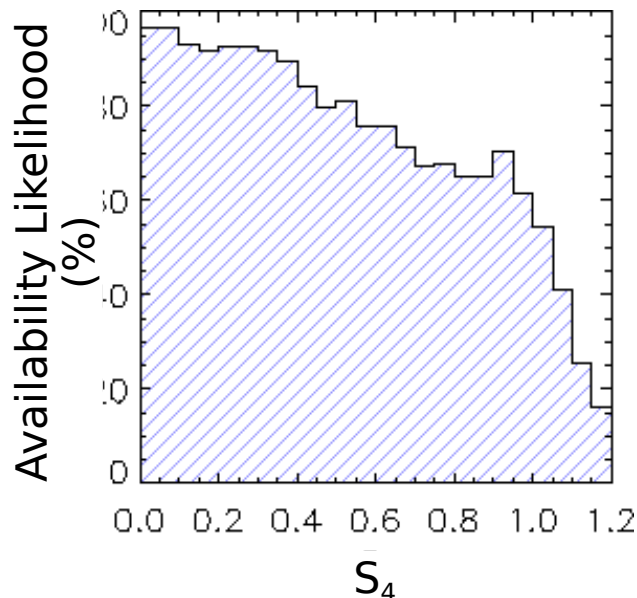
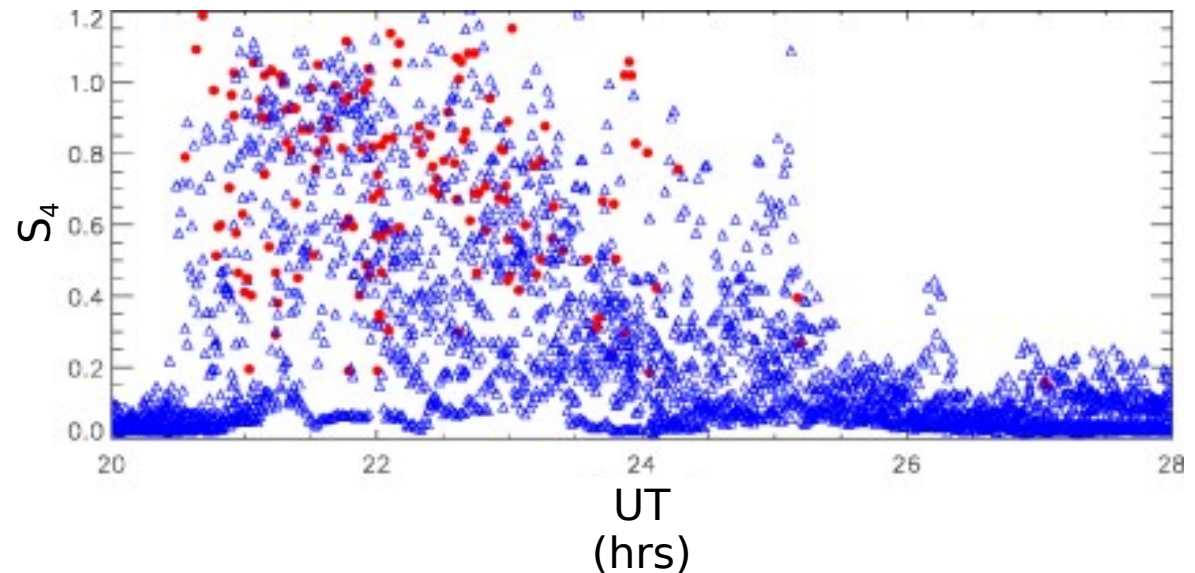
16 Mar 2002, ASI

Quality receivers report which satellites used in NAV

Example:

blue = used in NAV

red = not available  
(corresponds to spike in DOP)



Likelihood satellite will be available decreases as scintillation intensity increases. Each receiver type will have its own distribution.

Best metric might depend on receiver's "failure mode"

- If **fades** tend to break delay lock loop (DLL), use  $S_4$ .
- If due to **phase fluctuations** tend to break the phase lock loop (PLL), use  $\sigma_\phi$ .



# Simulating GPS Position Errors



Once we have modeled which satellites the receiver will track, we model the ranging errors and perform a standard navigation solution for the perturbed receiver position.

GPS range equation for each satellite,  $k$ :

$$P_{rs}^k + C_r + E_{rs}^k = \|\mathbf{R}_r - \mathbf{R}_s^k\|, \quad k=1, \dots, n$$

We model the  $k^{\text{th}}$  pseudorange:

$$\underbrace{P_{rs}^k}_{\text{modeled pseudorange}} = \underbrace{\|\mathbf{R}_r^0 - \mathbf{R}_s^k\|}_{\text{true range (via ephemeris)}} + \underbrace{[\gamma_s \hat{\phi}] S_4^k}_{\text{scintillation induced ranging error}}$$

Linearize the range equations about an initial estimate and solve by iteration:

$$\underbrace{\begin{bmatrix} (X_r - X_s^1)/R_{rs}^1 & (Y_r - Y_s^1)/R_{rs}^1 & (Z_r - Z_s^1)/R_{rs}^1 & (-1) \\ (X_r - X_s^2)/R_{rs}^2 & (Y_r - Y_s^2)/R_{rs}^2 & (Z_r - Z_s^2)/R_{rs}^2 & (-1) \\ \vdots & \vdots & \vdots & (-1) \\ (X_r - X_s^n)/R_{rs}^n & (Y_r - Y_s^n)/R_{rs}^n & (Z_r - Z_s^n)/R_{rs}^n & (-1) \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} dx \\ dy \\ dz \\ dc \end{bmatrix}}_{\mathbf{D}} = \underbrace{\begin{bmatrix} P_{rs}^1 - R_{rs}^1 \\ P_{rs}^2 - R_{rs}^2 \\ \vdots \\ P_{rs}^n - R_{rs}^n \end{bmatrix}}_{\mathbf{L}} \quad \text{where} \quad R_{rs}^k = \|\mathbf{R}_r - \mathbf{R}_s^k\|$$

Least squares solution to the over-determined system

$$\mathbf{A}\mathbf{D}=\mathbf{L} \text{ is } \mathbf{D}=(\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathbf{L}$$

Update the receiver position  $\mathbf{R}_r \rightarrow \mathbf{R}_r + [D[1], D[2], D[3]]^T$  and repeat until convergence.



# Application of the Model: Positioning Errors at Ascension Island

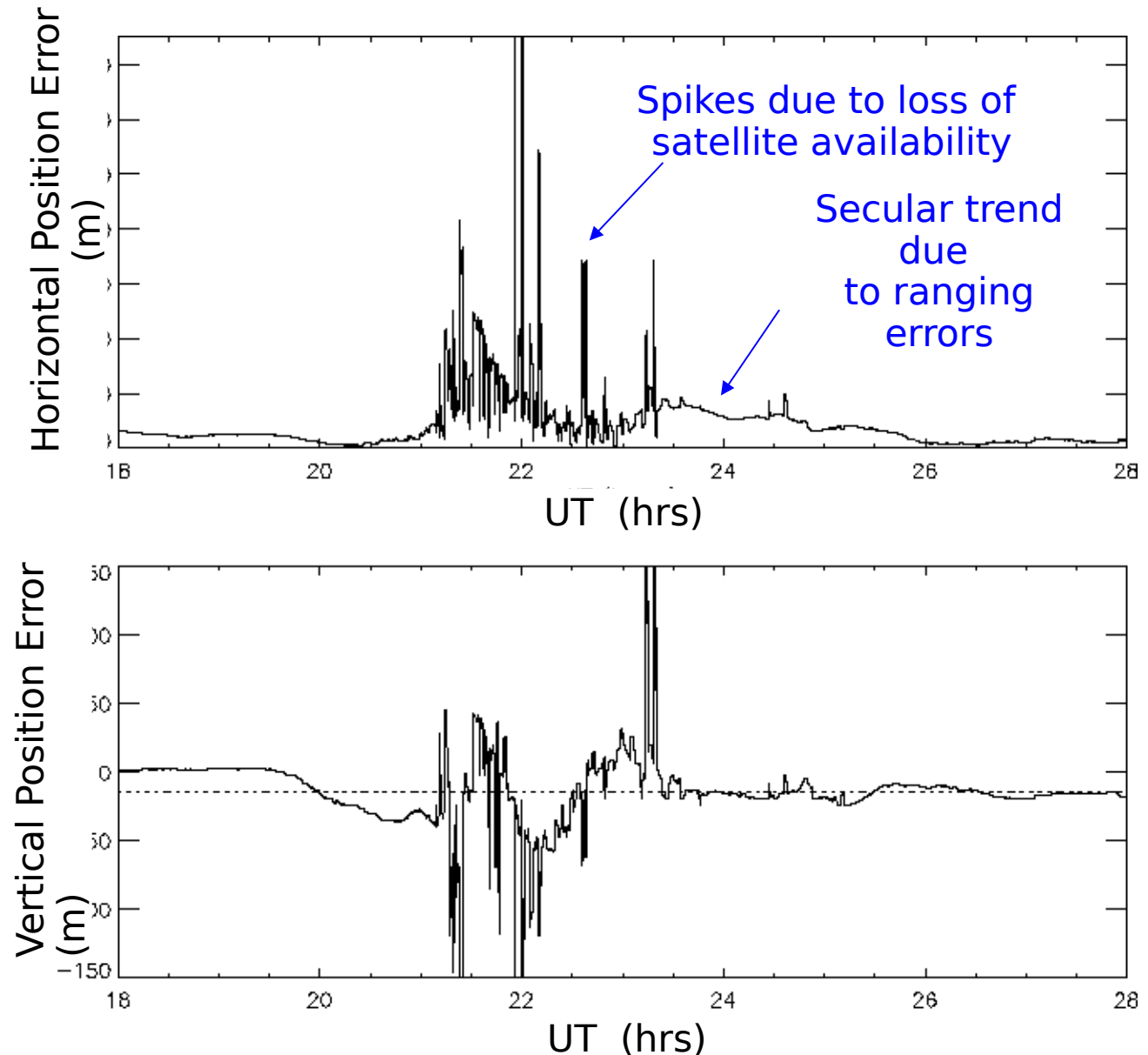


Actual positioning  
errors  
at ASI on 16 mar 2002

Goal:

Using only  
measurements  
of  $S_4$  and precise  
ephemeris, reproduce  
these position error  
results.

Only scintillation  
errors are included,  
assumes other effects  
negligible by  
comparison, including  
satellite and receiver  
clock errors,  
tropospheric errors,  
etc.





# Preliminary Simulation Results

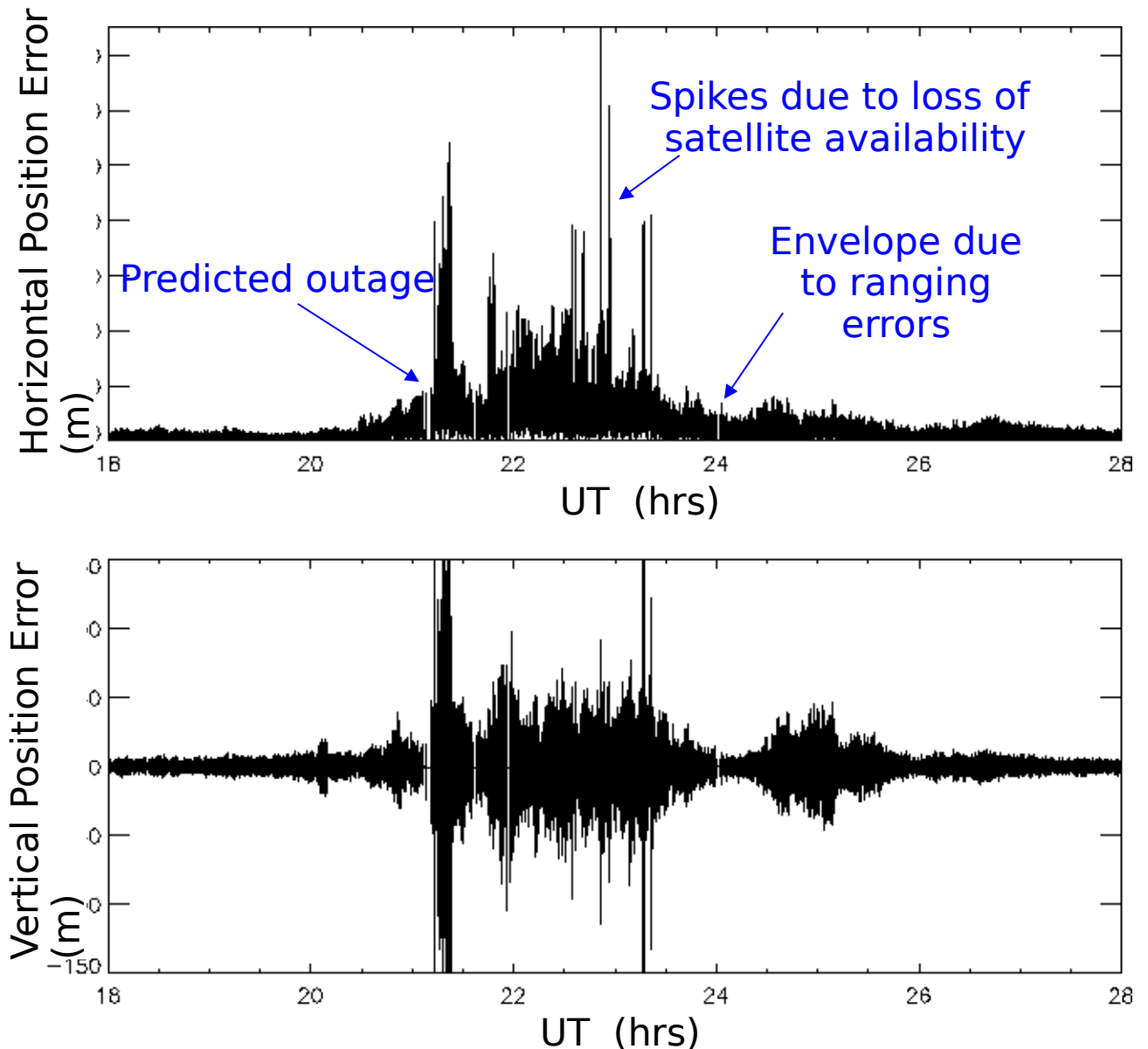


Simulation results using the scaling factor,  $\gamma_s = 70$  m

Explanation for rapid fluctuations:

Random range perturbations are not correlated in time, unlike in the real world

Even though we have an  $S_4$  measurement only once per minute, we evaluate the model every second so we can do statistics.





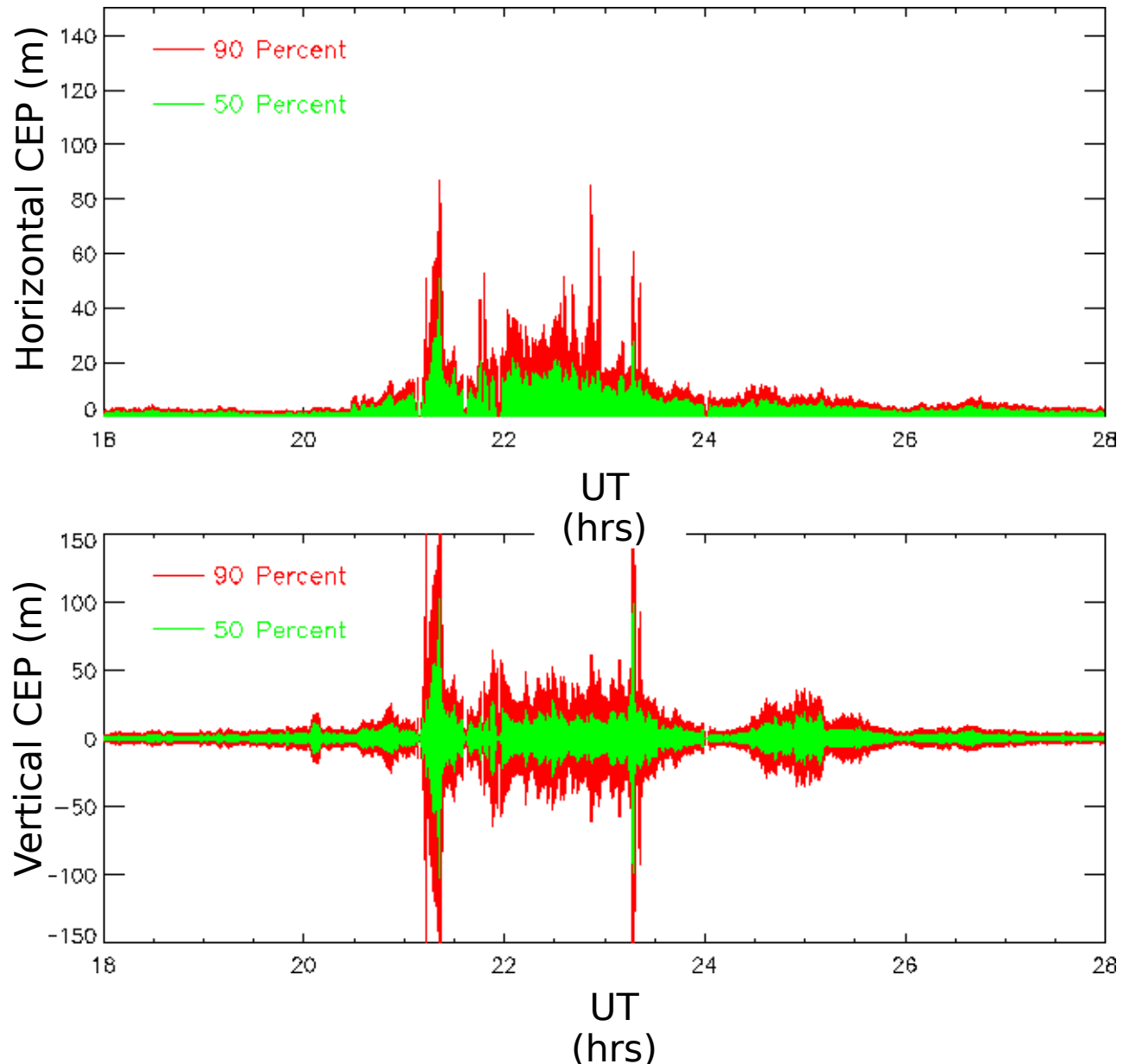


# Statistical Analysis of the Simulation Results



Sixty realizations per minute allow us to estimate CEP

We can also **invert** these statistics: e.g., for a given accuracy requirement, report the probability that this requirement is met. Coupled with a regional model of scintillation intensity (e.g., IONSCINT-G, model can generate **regional position error maps**







# Conclusions



- Ionospheric scintillation contributes to GPS positioning errors in two ways:
  - Intermittent link loss which degrades the effective constellation geometry
  - Ranging errors along each scintillating satellite-receiver link
- Model presented here couples both effects to statistically mimic scintillation induced errors
  - Simple parameters tunable for different receiver types
  - More sophisticated representations under investigation
- The model can predict (statistically) receiver outages and time-series of positioning errors from a single receiver; coupled with regional scintillation specification (i.e., SCINDA, IONSCINT) it can generate regional GPS error maps
- AFRL possesses extensive solar maximum data set of GPS performance during scintillation supporting unique capability for algorithm development



# Future Tasks

## Space Environment M&S



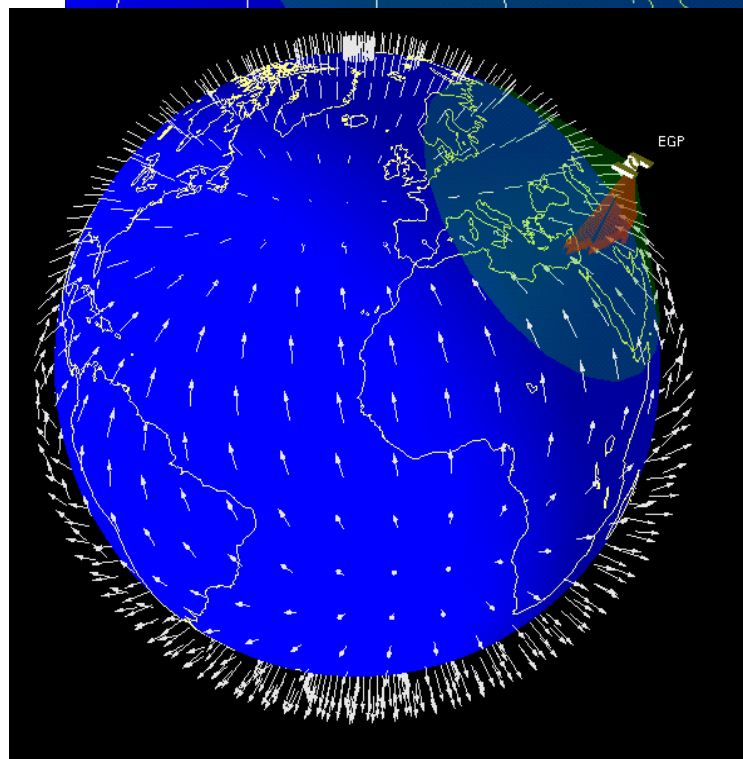
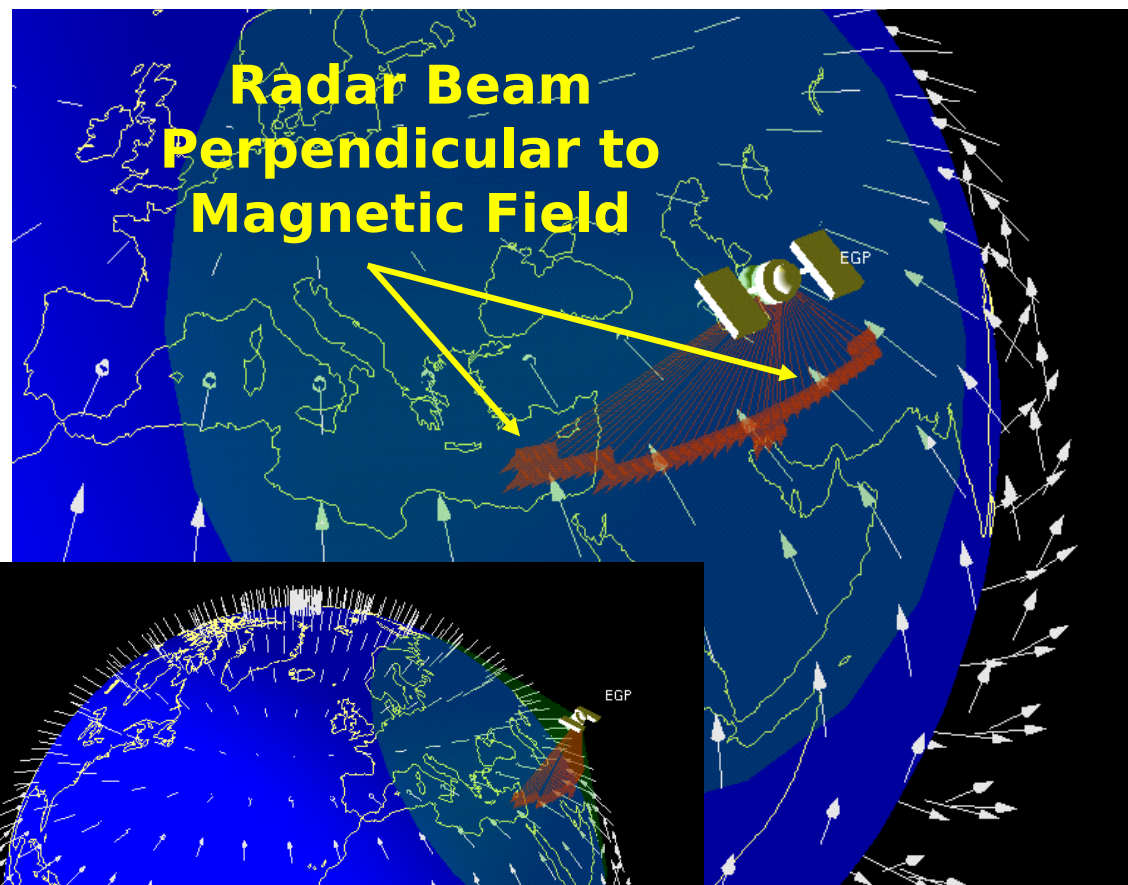
- Reasonable fidelity GPS navigation error tool
  - Impact still TBD; probably most significant unresolved space weather issue; requires simulation to address
  - Leverages on-going SMC and ICR&D project and FY04 investment
  - Questions should be answered well before next solar maximum (2010-2012)
- Include 'frequency selective' scintillation phenomena for wide band waveforms
  - Leverages current joint US-UK data collection project and past Space-based radar (SBR) simulation investments
  - Expands utility to ground and space-based radar surveillance communities
  - Include scintillation in existing SBR simulation (finite task)



# Ionospheric Effects on SBR Analysis & Simulation Tool



- SBR system susceptible to distributed clutter at high & low latitudes
  - Incorporating high latitude clutter model
  - Equatorial model in development
- Scintillation, TEC effects to be added
- Includes radar operation parameters and signal processing
- Maps response to frequency domain



**Quantitative Development Tool: Visualization of Clutter Regions Based on Modeled Magnetic Field Geometry**